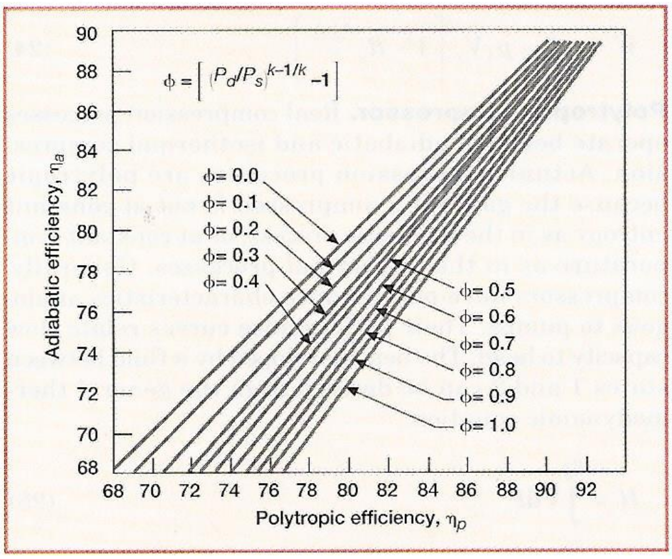

Fig. 4. Polytropic and adiabatic efficiency for a perfect gas ($Z = 1$).

$$p_1 V_1 = \frac{Z_1 R T_1}{M_w} \tag{32}$$

where:

Z_1 = compressibility factor at suction
T_1 = absolute temperature at suction, K
M_w = molecular weight, kg/kg mol
R = Gas constant, 8.314 kJ/kg mol K

Substituting Eq. 32 into Eq. 31, the polytropic head, H_p, becomes:

$$H_p = \frac{Z_1 R T_1}{M_w} \left(\frac{n}{n-1} \right) \left[R_c^{\frac{n-1}{n}} - 1 \right] \tag{33}$$

If the compressibility factor, Z_2, for the gas at discharge conditions is significantly different from that of the suction, then the average compressibility factor, Z_{avg}, is used to calculate the polytropic head.

Table 1. Polytropic efficiencies for various types of machines

Machine	Condition	E_p
Centrifugal compressor	Best	0.80
Centrifugal compressor	Fair	0.72
Reciprocating compressor	Best	1.00
Reciprocating compressor	Fair	0.92
Axial or rotary vane compressor	Best	0.92
Axial or rotary vane compressor	Fair	0.85
Rotary lobe compressor	Average	0.57
Small bore or pipeline reciprocating compressor	Good	1.05
Internal combustion engine	Good	1.45

Table 2. Approximate mechanical losses as a percentage of a gas power requirement

Gas power requirement		
English, hp	**Metric, kW**	**Mechanical losses, %**
0 to 3,000	0 to 2,500	3
3,000 to 6,000	2,500 to 5,000	2.5
6,000 to 10,000	5,000 to 7,500	2
10,000+	7,500+	1.5

$$Z_{avg} = \frac{Z_1 + Z_2}{2} \tag{34}$$

The polytropic head is defined by:

$$H_p = \left(\frac{Z_{avg} R T_1}{M_w} \right) \left(\frac{n}{n-1} \right) R_c^{\frac{n-1}{n}} - 1 \tag{35}$$

This can also be expressed as:

$$H_p = \left(\frac{8.314\, Z_{avg}\, T_1}{M_w} \right) \left(\frac{n}{n-1} \right) \left(R_c^{\frac{n-1}{n}} - 1 \right), \text{ kJ/kg} \tag{36}$$

M_w = molar mass of the gas, kg/kg mol
T_1 = suction temperature, K

The discharge temperature, t_2, is given by:

$$t_2 = \frac{(H_p)\,(M_w)}{(Z_{avg}\, R)} \left(\frac{n-1}{n} \right) + t_1 \tag{37}$$

There is a limit on the temperature such as in olefin or butadiene plants to prevent polymerization. At temperatures greater than 230°C to 260°C, the approximate mechanical limit, problems of sealing and casing growth could occur. High temperature requires a special and high cost machine. Therefore, multistage compressors are designed within the temperature range of 120°C to 150°C.[4]

In industrial compressors or expanders, the compression or expansion path will be polytropic. Therefore, the polytropic work produced (or required) can be derived from Eq. 18. That is:

$$(-W)_{poly} = \frac{n}{n-1} \{ p_2 V_2 - p_1 V_1 \} \tag{38}$$

since:

$p_1 V_1^n = p_2 V_2^n$ for polytropic condition,

Eq. 38 becomes:

$$(-W)_{poly} = \frac{n}{n-1} p_1 V_1 \left\{ \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right\} \tag{39}$$

where:

$$R_c = \frac{p_2}{p_1} \text{ and } p_1 V_1 = \frac{Z_1 R T_1}{M_w}$$

Eq. 39 can be further expressed to give

$$(-W)_{poly} = \frac{n}{n-1} \left(\frac{Z_1 R T_1}{M_w} \right) \left\{ R_c^{\frac{n-1}{n}} - 1 \right\}, \text{ kJ/kg} \tag{40}$$

The negative sign shows that the power is put into the system.

The actual work required is defined by:

$$(-W)_{actual} = \frac{(-W)_{poly}}{E_p}, \text{ kJ/kg} \tag{41}$$

The power required is: